

Research Article

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Monitoring bulk material pressure on bottom of storage using DEM

<https://doi.org/10.1515/eng-2019-0080>

Received Aug 26, 2019; accepted Oct 13, 2019

Abstract: This article focuses on the experimental verification of a mathematical derivation of Janssen's theory, which describes the distribution of pressures within the bulk material and pressure distribution on the walls of storage facilities. The experimental verification is performed in two ways. The first is the real measurement of the load transfer in a bulk material cylinder and the second is similar to the detection of the load transfer through simulations using the DEM method. The aim is to compare the results of the theoretical calculation according to Janssen's theory with the real measurement and a simulation of exactly the same model situation. At the beginning of any design or optimisation of existing transport or storage facilities, the most important is the analysis of the bulk material in the form of measurements of mechanical-physical properties. The analysis methods used are also described here. The pressure at the bottom of a storage container between the methods used showed negligible differences. From this finding, it can be concluded that the DEM method is a very suitable means for verifying the design of transport and storage facilities. The simulations provide important information and insights that can also be used to optimise existing transport or storage facilities.

Keywords: bulk material, storage, DEM

1 Introduction

In the past, designers had to rely solely on empirical calculations and their own experience in designing storage facilities. Currently, there are modern methods of construc-

tion, which verify the correct use of design, computer simulation and stress analysis. In this way, critical design errors that can cause irreversible deformations or the total collapse of the storage facilities in real operation are eliminated. On the other hand, it is possible to avoid the oversized construction of storage facilities using computer simulations. This research is focused on the method of determination of pressure inside storage facilities by means of mathematical derivation, DEM simulation and experimental verification. To determine the pressure in the bulk material cylinder and the pressure on the structure of storage facilities, it is not appropriate to use Pascal's law, which assumes a uniform spread of the pressure in all directions. Pascal's theory does not include any information about the structure of storage facilities and the stored material is only characterised by the density (bulk density for bulk materials) in the calculation of the pressure. It follows that the theory is applicable only for liquids and bulk materials with the angle of internal friction ϕ_e approaching 0° . In the past, Pascal's theory was also used for calculating the pressure in storage facilities for bulk materials with the angle of internal friction $\phi_e = (0^\circ, 90^\circ)$, resulting in the substantial oversizing of the proposed structures [1].

Pascal's theory can be expressed mathematically as follows:

$$\sigma_1 = \sigma_2 = \rho_s \cdot g \cdot h \quad (1)$$

where: σ_1 – vertical pressure within the bulk material [Pa]

σ_2 – horizontal pressure within the bulk material [Pa]

ρ_s – bulk density of the stored material [$\text{kg} \cdot \text{m}^{-3}$]

g – gravitational acceleration [$\text{m} \cdot \text{s}^{-2}$]

h – bulk material cylinder height [m]

A more accurate computation theory for determining pressures in bulk materials and pressures acting on the structure of storage facilities is Rankine's theory [1], which also contains a number of simplification conditions, e.g. the bulk material is incoherent, homogeneous, isotropic, perfectly drained, and neglects wall friction. Vertical pressure within the bulk material is defined similarly to Pascal's theory [1] as follows:

$$\sigma_1 = \rho_s \cdot g \cdot h \quad (2)$$

The horizontal pressure within the bulk material is specified by the fluidity coefficient k , which reflects the impor-

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tant material property – the angle of internal friction ϕ_e .

$$k = \frac{1 - \sin(\phi_e)}{1 + \sin(\phi_e)} \quad (3)$$

where

$$R = \frac{S}{o} \quad (4)$$

S – cross-sectional area of the storage facility at the site of the studied section [m^2]

o – circumference of the inner shell of the storage facility at the site of the studied section [m]

μ – coefficient of friction between the stored material and the inner surface of the storage space [-]

$$\mu = \tan \phi_w R \quad (5)$$

R – hydraulic radius [m]

k – fluidity coefficient [-]

ϕ_e – angle of internal friction [$^\circ$]

The horizontal pressure is given by the relation:

$$\sigma_2 = \sigma_1 \cdot k \quad (6)$$

The standardised theory to calculate the pressure in storage facilities is Janssen's theory [1–6]. The computing standard according to this theory has been incorporated into the following standard:

ČSN (Czech National Standard) 73 5570 Design of structures for storages, which is not currently effective and has been superseded by the standard ČSN EN 1991-4 ed. 2 Eurocode 1: Load on structures [7].

The standardised calculation according to Janssen's theory [1–7] includes information about the structure of the storage facility and also very important information about the stored material, which are mechanical and physical properties. The basic calculation of the horizontal pressure on the structure of the storage facility is given by the relation Eq. (7) [1].

$$p_h = \frac{\gamma \cdot R}{\mu} \cdot \left(1 - \frac{1}{e^{\frac{\mu \cdot K \cdot h}{R}}} \right) \gamma \quad (7)$$

γ – volumetric weight [$\text{N} \cdot \text{m}^{-3}$]

$$\gamma = \rho_s \cdot g \phi_w \quad (8)$$

ϕ_w – angle of external friction between the stored material and the inner surface of the storage space [$^\circ$]

K – lateral pressure ratio [-]

$$K = 1.1 \cdot (1 - \sin \phi_e) h \quad (9)$$

h – height of the material from the highest point of the storage facility to the examined cross-section [m]

The vertical pressure on the structure of the storage facility is given by the relation:

$$p_v = \frac{p_h}{K} \quad (10)$$

Shear stress acting in the vertical direction on the inner peripheral surface of the structure of the storage facility is given by:

$$p_w = p_h \cdot \mu \quad (11)$$

Figure 1 shows a container characterizing a storage device showing the pressures of bulk material acting on the walls and bottom of the storage device.

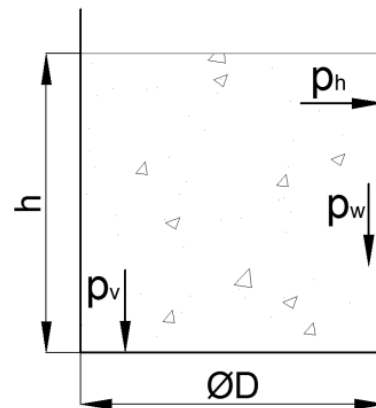


Figure 1: Pressures of the internal storage walls.

2 Materials and methods

2.1 Model

The calculation of the pressures acting on the inner surfaces of the storage is performed according to the mathematical model of Janssen's theory described above. Mathematical derivation is applied to the model storage, which is shown in Figure 2. The storage has a diameter $D = 0.1$ m and height $h = 0.44$ m. The internal surface of the storage is made from PVC material having surface roughness $Ra = 12.5 \mu\text{m}$.

2.2 Material

The stored material is represented by glass beads of the diameter $d = 3$ mm, which are poured into the storage container to fill it up completely. For the mathematical determination of the pressures on the structure of the stor-

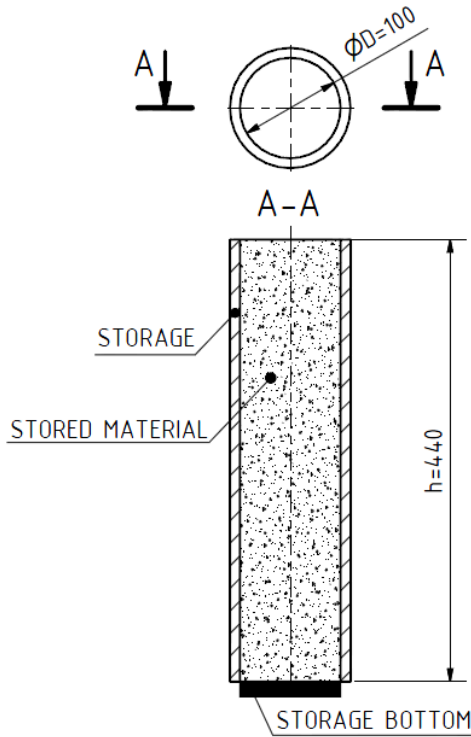


Figure 2: Diagram of the model storage



Figure 3: Image of the stored material

age container, it is necessary to know some basic mechanical and physical properties occurring in the samples of Janssen's theory. These are the bulk density, the wall friction angle between the stored material and the internal surface of the storage area, and the angle of internal friction. Figure 3 shows a sample of glass beads used in the experiment.

2.3 Bulk density ρ_s

The bulk density was measured using a simplified volume method. The bulk density is based on the known volume $V = 0.0005 \text{ m}^3$ of loosely poured glass beads and a known net weight of the given volume $m_{vk} = 0.798 \text{ kg}$. The measurement is repeated at least 10 times. The calculation of the bulk density is performed according to Eq. (12).

$$\rho_s = \frac{m_{vk}}{V} \quad (12)$$

The calculated result of the bulk density is $\rho_s = 1596 \text{ kg}\cdot\text{m}^3$.

2.4 Wall friction angle ϕ_w

The measurement of the wall friction angle was performed on a rectilinear Jenike shear machine. The wall friction angle determines the degree of the loss of work during the displacement of the bulk material across the solid contact material. The contact material in this case is especially hardened PVC material which constitutes the storage interior [8]. These values indicate the overcoming of the minimum degree of energy for material displacement across the used hardened PVC. The angle of wall friction between the glass beads of diameter $d = 3 \text{ mm}$ and the internal surface of the storage space, which is made of PVC material, the surface roughness $Ra = 12.5 \mu\text{m}$ was measured on the linear Jenike shear machine. The wall friction angle is $\phi_w = 13.1^\circ$.

2.5 Angle of internal friction φ_e

The angle of internal friction was measured on a Schulze ring shear tester. The principle of measurement of the angle of internal friction is the measurement of the time dependence of shear force which is necessary for the deformation of the bulk body in the shear chamber through a shear zone under normal load, for a given density of the bulk material. The density for a given measurement is achieved through consolidation (compaction) with a defined force load. The shear force is applied by rotating the device cell, and the torque is transmitted by two tie rods, which are fixed on the lid of the shear lid during the rotation measurement test, Schulze. The angle of internal friction determines the degree of internal work loss. During the displacement of the material, it affects the composition of the sample in terms of the representation of the individual fractions, particle shape, and their linkages [9, 10]. The angle of internal friction of glass beads of diameter $d = 3 \text{ mm}$ is $\varphi_e = 26^\circ$.

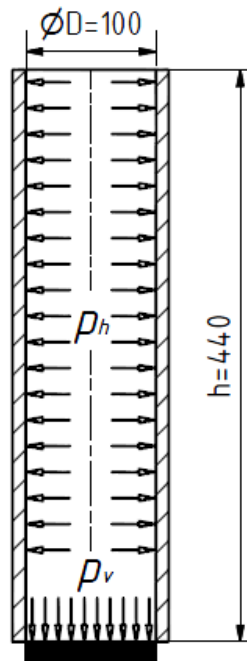


Figure 4: Image of pressures acting inside the storage

Table 1

h [mm]	p_h [Pa]	p_v [Pa]
0	0	0
40	347	560
80	624	1006
120	844	1361
160	1019	1644
200	1158	1868
240	1269	2047
280	1357	2189
320	1428	2303
360	1484	2394
400	1528	2465
440	1564	2523

2.6 Pressure in the storage

Inside the storage shown in Figure 4, the vertical pressure acts on the bottom of the storage container p_v and the horizontal pressure acts on the inner peripheral surface of the storage container p_h , which are the subject of the findings.

Table 1 shows the values of the pressures acting inside the storage for different depths of the material h .

Figure 5 shows the relation of the pressures acting on the inside walls of the storage. The figure shows the progress of the horizontal pressure p_h and the vertical pressure p_v . For comparison with the experimental part, we

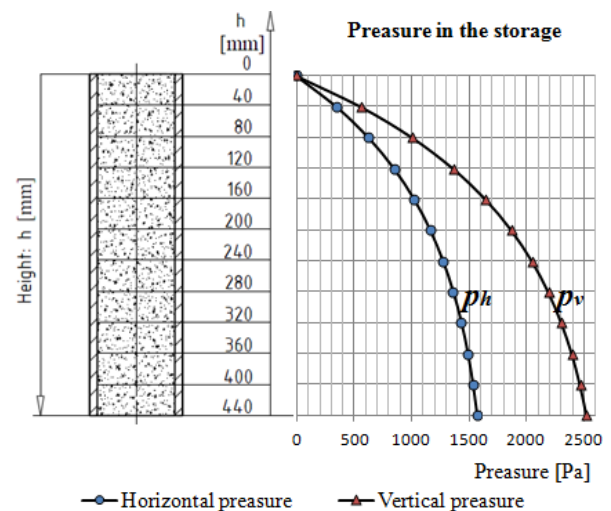


Figure 5: The theoretical relations of the pressure inside the storage

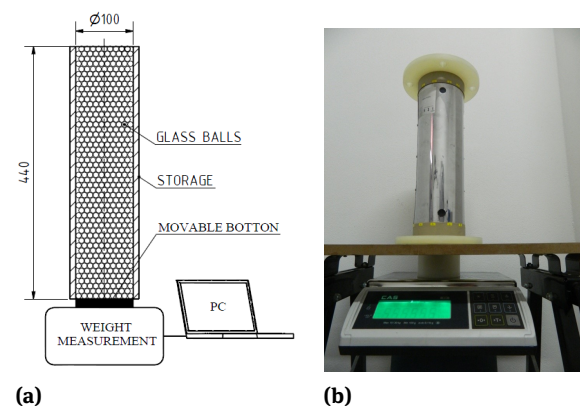


Figure 6: Diagram of the measurement stand: a) scheme of measurement stand, b) real measurement stand

will further be only interested in the maximum values of the horizontal and vertical pressure, calculated at the bottom of the storage container.

Horizontal pressure at the height $h = 0.44$ m: $p_h = 1564$ Pa

Vertical pressure at the height $h = 0.44$ m: $p_v = 2523$ Pa

2.7 Experiment

The experimental part focuses on measuring the actual pressure of the stored material, in this case glass beads of the diameter $d = 3$ mm acting on the internal structure of the storage container. The diagram of the measurement stand is shown in Figure 6 a) and the real measurement stand is shown in Figure 6 b). The experimental measure-

ment is focused only on the measurement of the vertical pressure p_v acting on the bottom of the storage container.

The measurement stand is composed of four basic parts: the storage container, movable bottom, measuring weight, computer.

The storage container is firmly attached to the base. A movable bottom is placed on the bottom edge of the storage container so that a minimum clearance is ensured between the inner edge of the storage container and the outer edge of the movable bottom. The clearance is set to prevent the stored glass beads of the diameter $d = 3$ mm from falling through. The movable bottom is placed on the weighing scales connected to the computer, which processes the measured values of the weight of the glass beads placed in the storage container and exerting pressure on the movable bottom. The diameter of the movable bottom is $D = 0.1$ m.

2.8 Measurement procedure

Glass beads of the diameter $d = 3$ mm were poured up to the upper edge of the prepared storage container with the volume $V_{exp} = 0.00345$ m³. The net weight of the poured amount of beads in the storage is $m_{V_{kexp}} = 5.518$ kg. After stabilizing the measured weight, the values were recorded on the PC. The whole procedure was repeated 10 times under the same conditions.

The bulk density calculated from the known net weight of the poured glass beads $m_{V_{kexp}}$ and the known storage volume V_{exp} is as follows:

$$\rho_{exp} = \frac{m_{V_{kexp}}}{V_{exp}} \quad (13)$$

The calculated result of the bulk density is $\rho_{exp} = 1599$ kg·m³.

A comparison of the bulk density $\rho_s = 1596$ kg·m³ from Eq. (12) and $\rho_{exp} = 1599$ kg·m³ from Eq. (13) shows a minimum difference in the calculated values. This inaccuracy arose when measuring the quantities needed for the calculation itself.

The resulting values of the measured weights m of the glass beads exerting pressure on the movable bottom of the storage container are given in Table 2.

The calculation of the maximum vertical pressure p_{vexp} acts on the movable bottom of the diameter $D = 0.1$ m according to Eq. (14). The maximum measured mass $m_2 = 0.199$ kg from Table 2 was used to calculate the maximum vertical pressure p_{vexp} .

$$p_{vexp} = \frac{m_2 \cdot g}{\frac{\pi \cdot D^2}{4}} \quad (14)$$

Table 2: Weight of the glass beads exerting pressure on the movable bottom

Number of measurement	Weight m [kg]	Average weight m_d [kg]
1	0.1935	0.1925
2	0.199	
3	0.195	
4	0.1935	
5	0.185	
6	0.195	
7	0.189	
8	0.197	
9	0.191	
10	0.187	

The calculated result of the vertical pressure is $p_{vexp} = 2485$ Pa.

2.9 Verification using DEM simulation method

The second verification of the mathematical derivation of *Janssen's theory* is performed using the DEM (Discrete Element Modelling) simulation method [12–19], which is based on calculating the mutual interactions of particles poured into the storage. Parameters such as, pressure, force, speed, torque, etc. can be obtained from DEM simulations. These parameters obtained from DEM simulations can advantageously be used and applied to other computational models of transport and storage applications such as, for example, a conveyor belt [20, 21]. There exists research on the issue of storage processes as published, for example, in the literature [14, 15]. There, the authors focused on the DEM modeling of the behavior of bulk materials when filling or emptying storage facilities. In these publications, dynamic changes in pressures within storage facilities were determined by DEM simulations, depending on the different operating conditions of the storage processes. Their DEM simulations use different combinations of contact models. In this case, the Hertz-Mindlin contact model was used to calculate the DEM simulation [22]. The actual simulation is carried out by means of the computer program EDEM Academic. Two basic elements are entered in the program. The first is a 3D model of the storage, which has exactly the same dimensions and physical characteristics as the real storage container used for the experimental measurements. A 3D storage model is shown in Figure 7a). The second dominant components entered in the

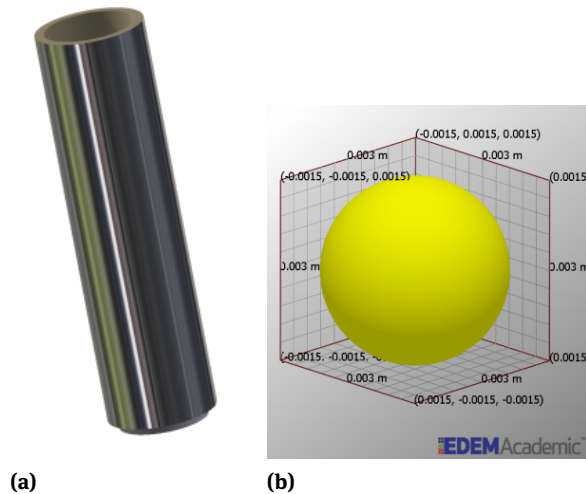


Figure 7: a) 3D model of the storage, b) model of the glass particle

Table 3: Input parameters for measuring forces in the EDEM Academic

	Contact material		
	PVC	Glass	Steel
Poisson's Ratio μ_1 [-]	0.39	0.23	0.3
Shear Modulus G [Pa]	$9 \cdot 10^8$	$3 \cdot 10^{10}$	$8.4 \cdot 10^{10}$
Density ρ [$\text{kg} \cdot \text{m}^{-3}$]	1780	2082	7850

Table 4: Input parameters detected by laboratory measurement for EDEM Academic

Interaction with:	Glass beads		
	PVC	Glass	Steel
Coefficient of restitution	0.93	0.5	0.5
Coefficient of static friction	0.32	0.58	0.3
Coefficient of rolling friction	0.004	0.01	0.01

simulation are the mechanical and physical properties of the glass beads of the diameter $d = 3$ mm. A model of a glass particle generated in the EDEM Academic program is shown in Figure 7b).

The material properties and interaction parameters were determined on the basis of measurements or taken from the literature [14, 23]. Table 3 presents the general material characteristics of the individual contact materials used in the experiment. Table 4 shows the parameters of interaction between the glass beads and the other contact materials. Throughout the experiment, the glass beads are in a static position. The interaction binding affecting the measurement results is only the coefficient of static friction between the glass beads and the PVC internal wall of the storage.

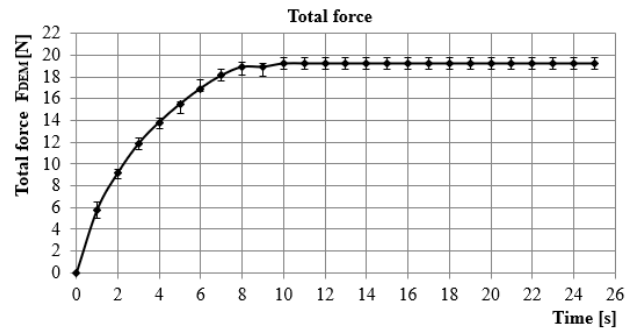


Figure 8: Dependence of the total force acting on the storage container bottom

Table 5: Comparison of vertical pressure acting on the storage container bottom

	p_v	p_{vexp}	p_{vDEM}
Vertical pressure [Pa]	2523	2485	2444

The generated particles in the storage model with a closed bottom fill the storage container to the upper edge. Full storage contains 152 699 glass beads of the diameter $d = 3$ mm. The total force acting on the bottom of the storage container is derived from the weight of the glass beads placed in the inner storage space. This average value of the total force was subtracted from the performed DEM simulations from a total of ten repetitions. The average value of the total force acting on the bottom of the storage is $F_{DEM} = 19.2$ N. The steady dependence of the total force acting on the storage container bottom is shown in Figure 8. The picture shows that filling the storage container took approximately 10 s.

Calculation of maximum vertical pressure p_{vDEM} acting on the storage container bottom of the diameter $D = 0.1$ m.

$$p_{vDEM} = \frac{F_{DEM}}{\frac{\pi \cdot D^2}{4}} \quad (15)$$

The calculated result of the vertical pressure is $p_{vDEM} = 2444$ Pa.

Table 5 compares the vertical pressure values obtained using three methods: Janssen's theory calculation, real experiment and DEM simulation. The comparison values show a high similarity, which confirms the fact that the DEM simulation method can be considered to be a reliable tool in the design of not only storage facilities. It should be noted that the accuracy of the DEM simulation and the plausibility of the results depend mainly on the accuracy of the input parameters entering the simulation environment, such as the material and physical properties of the

model, the mechanical-physical properties of the bulk material and the interaction parameters.

3 Conclusion

In this work, a DEM simulation and experiment were used to verify the mathematical derivation of Janssen's theory. In the first step, waveforms of pressures acting on the inside walls of a storage container were calculated using Janssen's derivation. The vertical pressure acting on the bottom of the storage container according to the calculation is $p_v = 2523$ Pa and the horizontal pressure exerted on the inside wall of the storage at the lowest point, which is at the depth $h = 0.44$ m, is $p_h = 1564$ Pa. This was followed by a practical experiment on a real storage container, monitoring only the vertical component of the pressure p_{vexp} acting on the bottom of the storage. The value of the maximum pressure is $p_{vexp} = 2485$ Pa. The second verification of Janssen's theory was performed by means of a computer simulation using the DEM method in the EDEM Academic program. The resulting vertical pressure acting on the bottom of the storage corresponds to $p_{vDEM} = 2444$ Pa. The results of the individual methods showed minimum differences. The resulting differences may be due to inaccuracies in the performance of the experiments or in the determination of the input parameters for the DEM method. It can be stated that the DEM method is a very useful tool for the verification of the design of transport or storage facilities. By verifying the design with DEM, it is possible to detect hidden defects or non-functional parts of the technology before an expensive prototype is manufactured.

Acknowledgement: This paper was conducted within the framework of the projects LO1404: Sustainable development of ENET Centre, Innovative and additive manufacturing technology - new technological solutions for 3D printing of metals and composite materials, reg. no. CZ.02.1.01/0.0/0.0/17_049/0008407 financed by Structural Funds of the European Union SP2019/115: Research in the optimised design of transport and storage processes using test equipment and simulation techniques and the Faculty of Mechanical Engineering.

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